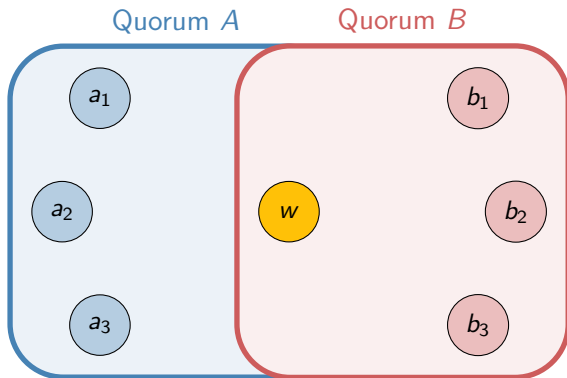


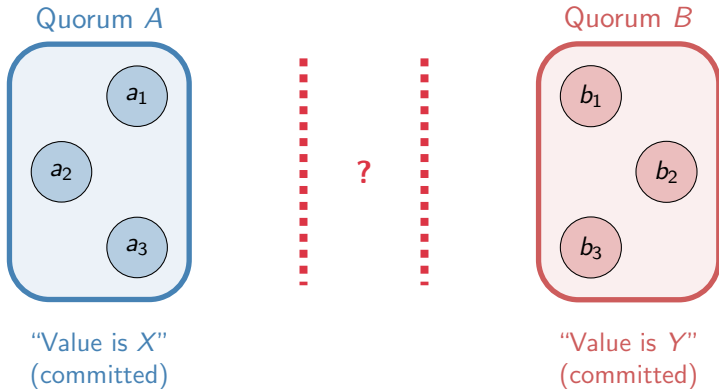
Why Consensus Works: The Intersection Property



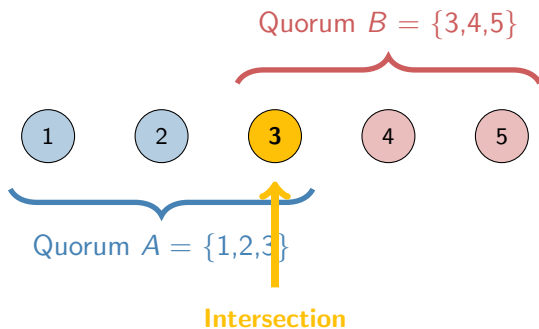
Node w witnesses both operations.

If I write to A and you read from B ,
node w ensures you see my write.

Why Consensus Breaks: Intersection Failure



Majority Quorums: Why $\lfloor n/2 \rfloor + 1$ Works



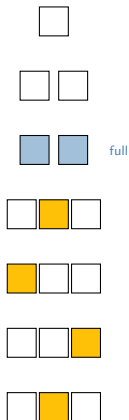
With $n = 5$ nodes, any majority quorum has ≥ 3 nodes.

Pigeonhole principle: Two sets of size 3 from 5 elements *must* share at least one element.

$$|A| + |B| > n \implies A \cap B \neq \emptyset$$

Crumbling Walls: Intersection with $O(\log n)$ Quorums

Quorum A

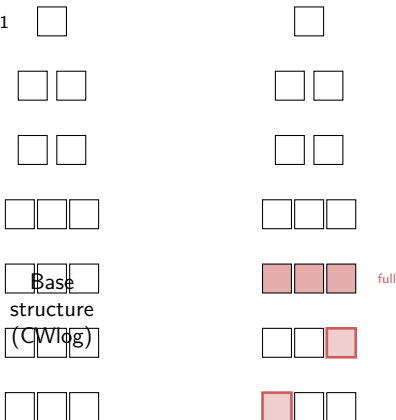


Quorum A (row 3): full row + representative from each row below

Quorum B (row 5): full row + representative from each row below

Intersection: B's full row is below A's, so A has a rep in row 5 — *inside* B.

Quorum B



Base
structure
(CWlog)

Recovery: The Path from S to S'

The witness knows:
— what was written
— in what order
— by whom



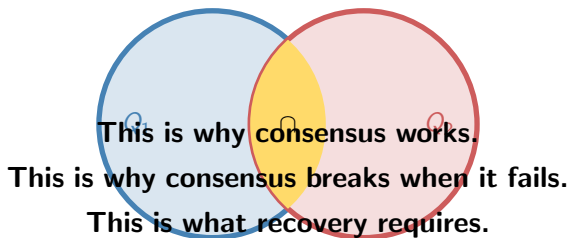
Nodes disagree
No single "truth"
Operations lost?

Recovery is possible if and only if
there exists a node that participated in *all* relevant operations

The intersection property guarantees such a node exists
That node can *arbitrate* the correct history

The Core Insight

A **quorum system** is any collection of sets where every pair shares at least one element.



Everything else is engineering.